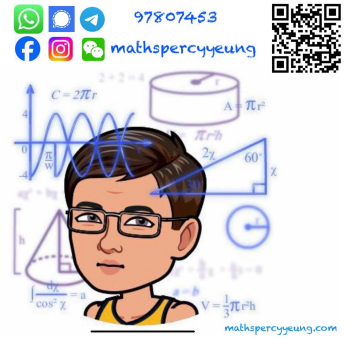
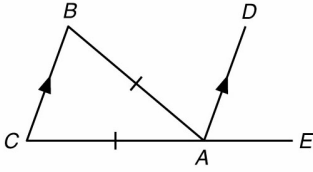
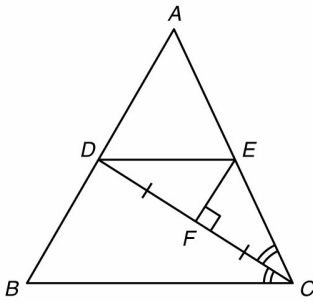


# Chapter 10 More about Deductive Geometry Set 1

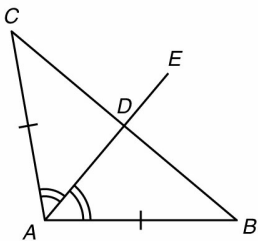
In the figure,  $AB = AC$  and  $DA \parallel BC$ . Prove that  $DA$  is the angle bisector of  $\angle BAE$ .



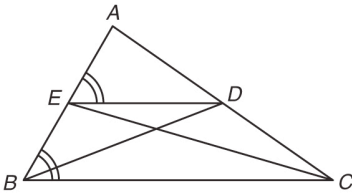
In the figure,  $CD$  is the angle bisector of  $\angle ACB$ .  $F$  is the mid-point of  $DC$  and  $EF \perp DC$ . Prove that  $DE \parallel BC$ .



In the figure,  $AB = AC$  and  $DA$  is the angle bisector of  $\angle BAC$ . Prove that  $ADE$  is the perpendicular bisector of  $BC$ .



In the figure,  $\triangle ABC$  is a triangle where  $BD$  is the median of  $AC$  and  $\angle AED = \angle ABC$ . Prove that  $CE$  is the median of  $AB$ .



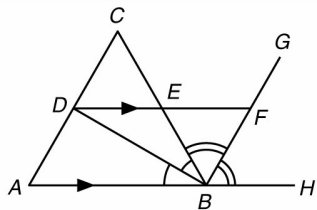
Determine whether each of the following sets of line segments can form a triangle. Briefly explain your answer.

- (a) 6 cm, 7 cm, 2 cm
- (b) 10 cm, 6 cm, 3 cm
- (c) 9 cm, 14 cm, 5 cm

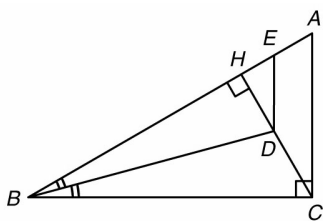
The lengths of the two sides of an isosceles triangle are 18 cm and 41 cm respectively.

- (a) Find the perimeter of the triangle.
- (b) Find the area of the triangle.

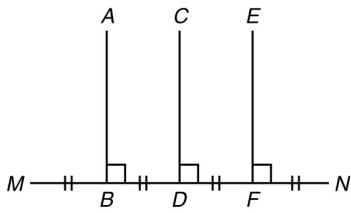
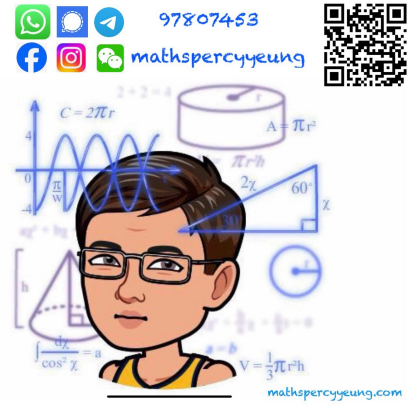
In the figure,  $BD$  and  $BG$  are the angle bisectors of  $\angle ABC$  and  $\angle CBH$  respectively. A line segment  $DF$  is drawn parallel to  $AB$ , such that it meets  $BC$  and  $BG$  at  $E$  and  $F$  respectively. Prove that  $DE = EF$ .



In the figure,  $\triangle ABC$  is a triangle where  $CH$  is the altitude of  $AB$  and  $\angle ACB = 90^\circ$ .  $BD$  is the angle bisector of  $\angle ABC$ , which meets  $CH$  at  $D$ .  $E$  is a point on  $AB$ , such that  $BE = BC$ . Prove that  $ED \parallel AC$ .

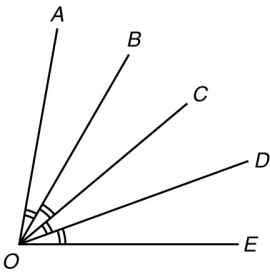


In  $\triangle ABC$ ,  $AB = 6$  cm,  $BC = 4.2$  cm and  $AC = x$  cm where  $x$  is an integer. Find the maximum and minimum values of  $x$ .



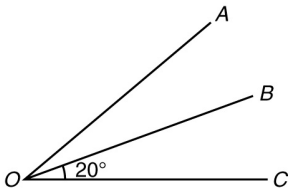
Referring to the figure, which line segment is the perpendicular bisector of

- (a)  $MD$  ?
- (b)  $MN$  ?
- (c)  $BF$  ?
- (d)  $DN$  ?

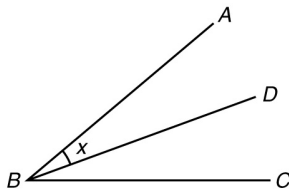


Referring to the figure, which line segment is the angle bisector of

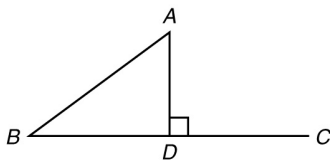
- (a)  $\angle AOC$  ?
- (b)  $\angle BOD$  ?
- (c)  $\angle COE$  ?
- (d)  $\angle AOE$  ?



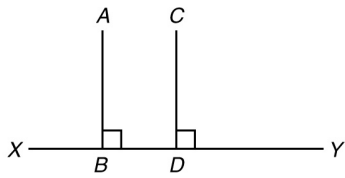
In the figure,  $OB$  is the angle bisector of  $\angle AOC$ . If  $\angle BOC = 20^\circ$ , find  $\angle AOC$ .



In the figure,  $BD$  is the angle bisector of  $\angle ABC$ . If  $\angle ABC = 50^\circ$ , find  $x$ .

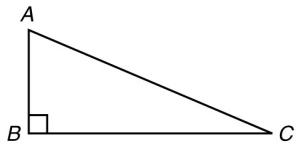


In the figure,  $AD$  is the perpendicular bisector of  $BC$ . If  $BC = 8$  cm and  $AD = 3$  cm, find the length of  $AB$ .

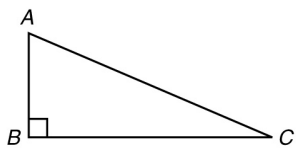


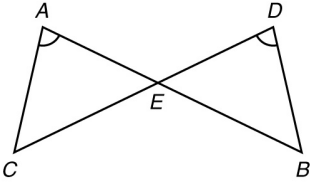
$AB$  and  $CD$  are the perpendicular bisectors of  $XD$  and  $XY$  respectively. Find  $XB : XY$ .

The figure shows  $\triangle ABC$  with  $\angle ABC = 90^\circ$ . Use straight edge and compasses to construct the angle bisector of  $\angle ABC$ .

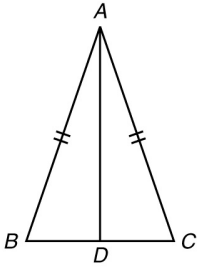


The figure shows  $\triangle ABC$  with  $\angle ABC = 90^\circ$ . Use straight edge and compasses to construct the perpendicular bisector of  $BC$ .

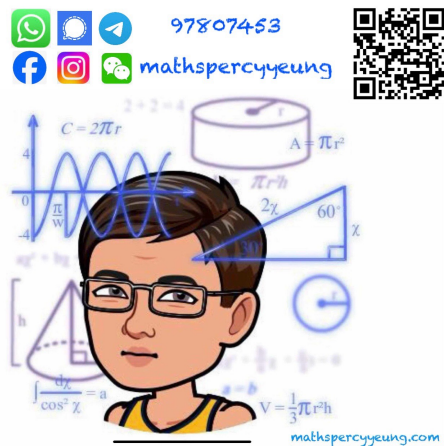




In the figure,  $AB$  intersects  $CD$  at  $E$ . If  $\angle CAE = \angle BDE$ , prove that  $\angle DBE = \angle ACE$ .

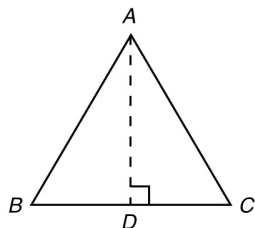


In the figure,  $AD$  is the angle bisector of  $\angle BAC$ . If  $AB = AC$ , prove that  $\triangle ABD \cong \triangle ACD$ .



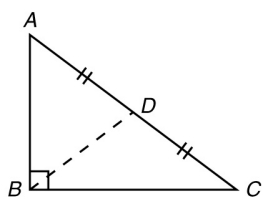
In each of the following triangles, write down the name of the dotted line.

(a)



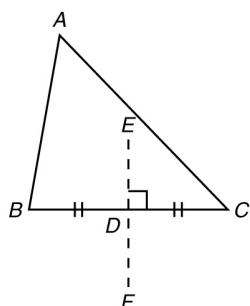
$AD$  is the \_\_\_\_\_  
of  $BC$  in  $\triangle ABC$ .

(b)



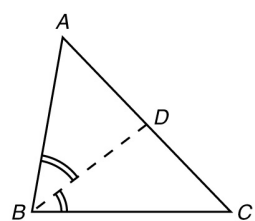
$BD$  is the \_\_\_\_\_  
of  $AC$  in  $\triangle ABC$ .

(c)



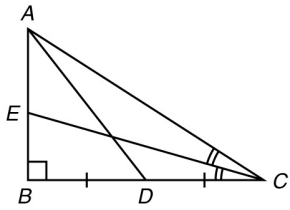
$EF$  is the \_\_\_\_\_  
of  $BC$  in  $\triangle ABC$ .

(d)



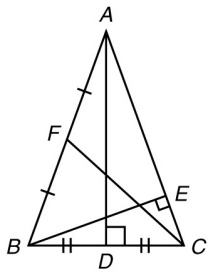
$BD$  is the \_\_\_\_\_  
of  $\angle ABC$  in  $\triangle ABC$ .





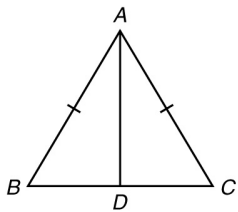
Referring to the figure, which line segment is

- (a) a median of  $\triangle ABC$ ?
- (b) an angle bisector of  $\triangle ABC$ ?

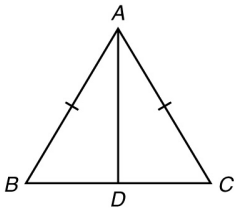


Referring to the figure, which line segment(s) is a/are

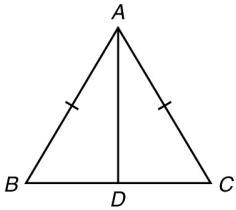
- (a) perpendicular bisector(s) of  $\triangle ABC$ ?
- (b) altitude(s) of  $\triangle ABC$ ?



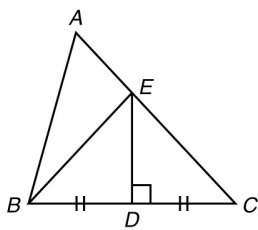
In the figure,  $AD$  is the median of  $BC$  in  $\triangle ABC$ . If  $AB = AC$ , show that  $\triangle ABD \cong \triangle ACD$ .



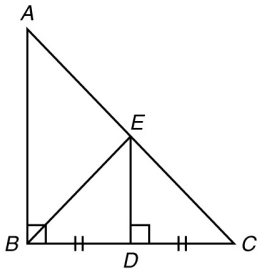
In the figure,  $AD$  is the altitude of  $BC$  in  $\triangle ABC$ . If  $AB = AC$ , show that  $\triangle ABD \cong \triangle ACD$ .



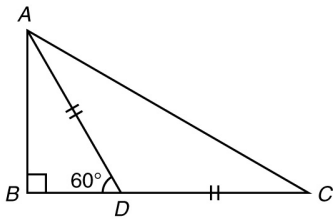
In the figure,  $AD$  is the angle bisector of  $\angle BAC$  in  $\triangle ABC$ . If  $AB = AC$ , show that  $\triangle ABD \cong \triangle ACD$ .



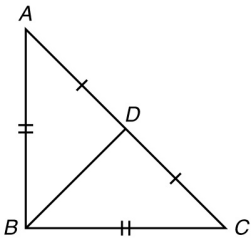
In  $\triangle ABC$ ,  $DE$  is the perpendicular bisector of  $BC$ . If  $DE = BD$ , show that  $BE$  is the altitude of  $AC$  in  $\triangle ABC$ .



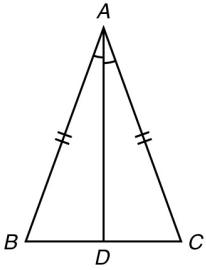
In the figure,  $DE$  is the perpendicular bisector of  $BC$  in  $\triangle BCE$ . If  $\angle ABC = 90^\circ$ , prove that  $BE$  is the median of  $AC$  in  $\triangle ABC$ .



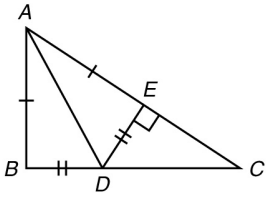
The figure shows a right-angled triangle  $ABC$ , where  $\angle B = 90^\circ$ . If  $AD = CD$  and  $\angle ADB = 60^\circ$ , prove that  $AD$  is the angle bisector of  $\angle BAC$  in  $\triangle ABC$ .



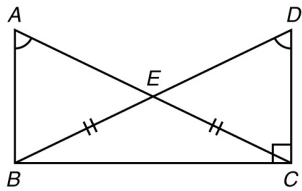
In the figure,  $BD$  is the median of  $AC$  in  $\triangle ABC$ . If  $AB = BC$ , show that  $BD$  is also the altitude of  $AC$  in  $\triangle ABC$ .



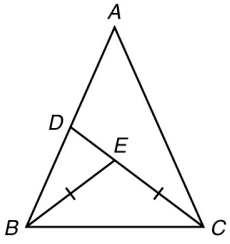
In the figure,  $AD$  is the angle bisector of  $\angle BAC$  in  $\triangle ABC$ . If  $AB = AC$ , prove that  $AD$  is also the perpendicular bisector of  $BC$  in  $\triangle ABC$ .



In the figure,  $DE$  is the altitude of  $AC$  in  $\triangle ACD$ . If  $AB = AE$  and  $DB = DE$ , prove that  $AB$  is an altitude of  $\triangle ABC$ .



In the figure,  $CD$  is the altitude of  $BC$  in  $\triangle BCD$ ,  $\angle BAC = \angle CDB$  and  $EB = EC$ . Prove that  $AB$  is the altitude of  $BC$  in  $\triangle ABC$ .



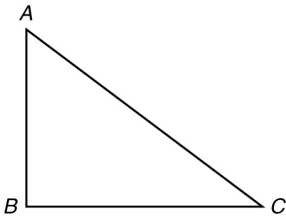
In the figure,  $AB = AC$  and  $EB = EC$ . If  $CD$  is the angle bisector of  $\angle ACB$  in  $\triangle ABC$ , prove that  $BE$  is the angle bisector of  $\angle DBC$  in  $\triangle BCD$ .

Determine whether each of the following sets of line segments can form a triangle. Briefly explain your answer.

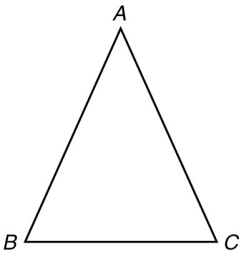
- (a) 3 cm, 5 cm, 6 cm
- (b) 2 cm, 5 cm, 8 cm
- (c) 5 cm, 12 cm, 13 cm

If the lengths of two sides of an isosceles triangle are 4 cm and 8 cm, what is the perimeter of the triangle?

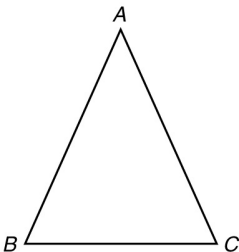
By using compasses and straight edge, locate the centroid of  $\triangle ABC$ .



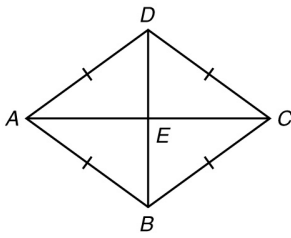
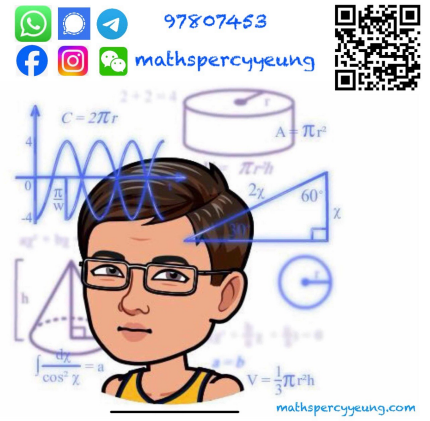
By using compasses and straight edge, locate the incentre of  $\triangle ABC$ .



By using compasses and straight edge, locate the orthocentre of  $\triangle ABC$ .

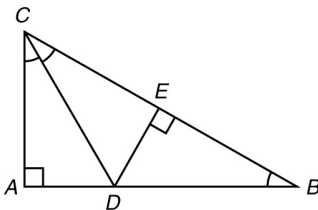






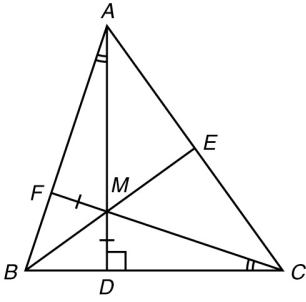
In the figure,  $AB = BC = CD = DA$ . Prove that

- (a)  $\triangle ABD \cong \triangle CBD$  and  $\triangle ABE \cong \triangle CBE$ ,
- (b)  $BD$  is the perpendicular bisector of  $AC$ .



In the figure,  $CD$  is the angle bisector of  $\angle ACB$  in  $\triangle ABC$ .  $\angle DBC = \angle DCB$  and  $\angle CAB = \angle DEB = 90^\circ$ .

- (a) Show that  $\triangle ACD \cong \triangle ECD$ .
- (b) Show that  $\triangle ECD \cong \triangle EBD$ .
- (c) Hence, find the ratio of the area of  $\triangle EBD$  to the area of  $\triangle ABC$ .

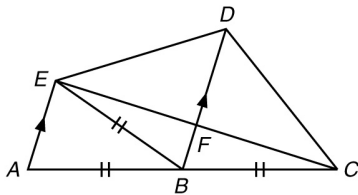


In the figure,  $AD$  is the altitude of  $BC$  in  $\triangle ABC$ ,  $\angle BAD = \angle BCF$  and  $MF = MD$ .  $AD$  intersects  $CF$  at  $M$  and  $BME$  is a straight line.

(a) Show that  $CF$  is the altitude of  $AB$  in  $\triangle ABC$ .

(b) Show that  $\triangle BMF \cong \triangle BMD$ .

Hence, deduce that  $BE$  is the angle bisector of  $\angle ABC$  in  $\triangle ABC$ .

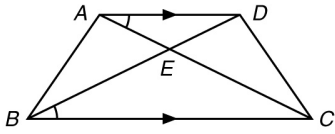


In the figure,  $AB = BC = BE$ ,  $AE \parallel BD$  and  $CE$  intersects  $BD$  at  $F$ .

(a) Prove that  $AE$  is the altitude of  $CE$  in  $\triangle ACE$ .

(b) Prove that  $\triangle BFE \cong \triangle BFC$ .

Hence, deduce that  $DF$  is the perpendicular bisector of  $CE$  in  $\triangle CDE$ .



The figure shows a quadrilateral  $ABCD$ .  $\angle DAC = \angle DBC$ ,  $AD \parallel BC$  and  $AC$  intersects  $BD$  at  $E$ .

(a) Prove that  $\triangle ABE \cong \triangle DCE$ .

(b) If  $AE$  is the median of  $BD$  in  $\triangle ABD$ , show that  $ABCD$  is a rectangle.

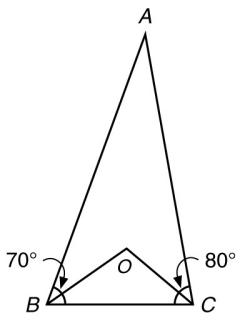
The perimeter of an isosceles triangle is 28 cm. If the length of one side is 8 cm, find the possible lengths of the other two sides.

The perimeter of an isosceles triangle is 36 cm. If the length of one side is 16 cm, find the possible lengths of the other two sides.

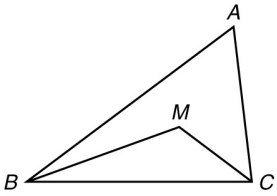
The perimeter of an isosceles triangle is 14 cm. If the length of one side is 3 cm, find the possible lengths of the other two sides.

In  $\triangle ABC$ ,  $AB = 5$  cm,  $BC = a$  cm and  $CA = b$  cm, where  $a$  and  $b$  are positive integers. If  $a + b = 7$  and  $a > b$ , find the possible values of  $a$  and  $b$ .

In  $\triangle PQR$ ,  $PQ = 5$  cm,  $QR = x$  cm and  $RP = y$  cm, where  $x$  and  $y$  are positive integers. If  $x + y = 15$  and  $x > y$ , find the possible values of  $x$  and  $y$ .



In the figure,  $\angle ABC = 70^\circ$  and  $\angle ACB = 80^\circ$ . If  $O$  is the incentre of  $\triangle ABC$ , find  $\angle BOC$ .

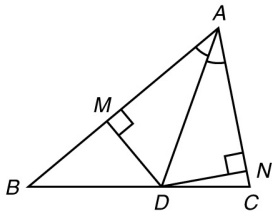
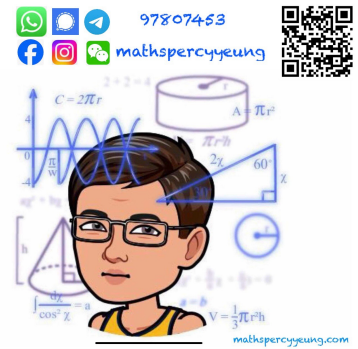


In  $\triangle ABC$ , the angle bisectors of  $\angle ABC$  and  $\angle ACB$  intersect at  $M$  and  $\angle BMC = 2\angle BAC$ .

(a) Show that  $\angle ABM + \angle ACM = \angle BAC$ .

(b) Find  $\angle BAC$ .

By using compasses and straight edge, construct a right-angled triangle  $ABC$ , where  $\angle B = 90^\circ$ ,  $BC = 4$  cm and  $AB = 3$  cm, and locate the centroid  $G$  of the triangle. Describe briefly the steps of construction.

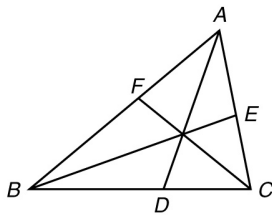


In the figure,  $AD$  is the angle bisector of  $\angle BAC$  in  $\triangle ABC$ .  $MD$  is the altitude of  $AB$  in  $\triangle ABD$  and  $ND$  is the altitude of  $AC$  in  $\triangle ACD$ .

(a) Show that  $\triangle AMD \cong \triangle AND$ .

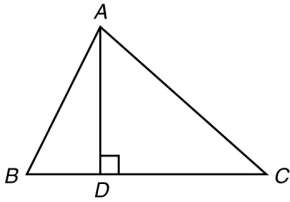
(b) Show that  $\frac{\text{area of } \triangle ABD}{\text{area of } \triangle ACD} = \frac{BD}{DC}$ . Hence, show that  $\frac{BD}{DC} = \frac{AB}{AC}$ .

(c)  $BE$  and  $CF$  are the other two angle bisectors of  $\triangle ABC$  as shown below.



By using the result of (b), express  $\frac{AF}{FB}$  and  $\frac{CE}{EA}$  in terms of  $AB$ ,  $AC$  and  $BC$ .

(d) Find the value of  $\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA}$ .

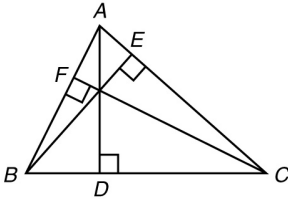


In the figure,  $AD$  is the altitude of  $BC$  in  $\triangle ABC$ .

(a) (i) Show that  $\frac{DC}{AD} = \frac{1}{\tan \angle BCA}$ .

(ii) Hence, show that  $\frac{BD}{DC} = \frac{\tan \angle BCA}{\tan \angle ABC}$ .

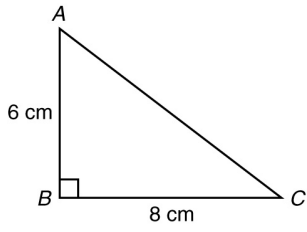
(b)  $BE$  and  $CF$  are the other two altitudes of  $\triangle ABC$  as shown below.



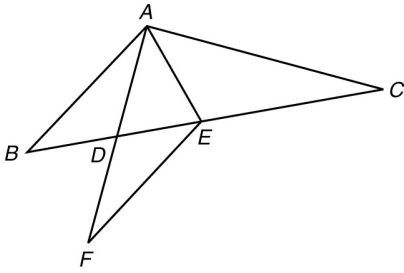
By using the result of (a), express  $\frac{AF}{FB}$  and  $\frac{CE}{EA}$  in terms of  $\tan \angle ABC$ ,  $\tan \angle BCA$  and  $\tan \angle CAB$ .

(c) Find the value of  $\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA}$ .



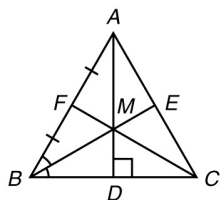
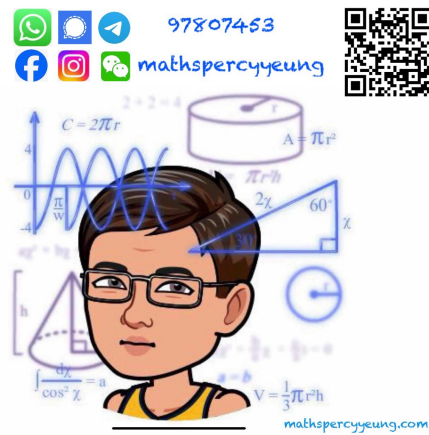


In the figure,  $AB = 6$  cm,  $BC = 8$  cm and  $\angle ABC = 90^\circ$ . Find the radius of the circumscribed circle of  $\triangle ABC$ .



In the figure,  $AD$  is the median of  $BE$  in  $\triangle ABE$ ,  $AE$  is the median of  $BC$  in  $\triangle ABC$  and  $DE$  is the median of  $AF$  in  $\triangle AFE$ . If  $2AB = BC$ , show that

- (a)  $\triangle ABE$  is an isosceles triangle,
- (b)  $\triangle DAB \cong \triangle DFE$ ,
- (c)  $\angle AEC = \angle AEF$ ,
- (d)  $AE$  is the angle bisector of  $\angle CAD$  in  $\triangle ADC$ .



In the figure,  $\triangle ABC$  is an equilateral triangle.  $AD$  is the altitude of  $BC$ ,  $BE$  is the angle bisector of  $\angle ABC$  and  $CF$  is the median of  $AB$ . If  $AD$ ,  $BE$  and  $CF$  intersect at  $M$ , which of the following statement(s) is/are true?

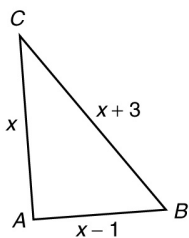
- I.  $M$  is the centroid.
- II.  $M$  is the incentre.
- III.  $M$  is the circumcentre.
- IV.  $M$  is the orthocentre.

- A. I and II only
- B. I, II and III only
- C. I, II, III and IV
- D. none of the above

Which of the following sets of line segments can form a triangle?

- I. 2 cm, 3 cm, 4 cm
- II. 4 cm, 6 cm, 8 cm
- III. 3 cm, 12 cm, 16 cm

- A. I. only
- B. II. only
- C. I and II only
- D. I, II and III



In  $\triangle ABC$ ,  $AC = x$ ,  $AB = x - 1$  and  $BC = x + 3$ . Which of the following must be true?

- A.  $x$  is an integer.
- B.  $x > 4$
- C.  $0 < x < 4$
- D. none of the above

The lengths of the three line segments of a triangle are 4 cm,  $x$  cm and  $y$  cm. Which of the following must be false?

- A. If  $x > y > 4$ , then  $4 + y > x$ .
- B. If  $y > 4 > x$ , then  $4 > y - x$ .
- C. If  $4 > x > y$ , then  $4 - x < y$ .
- D. If  $4 > x$  and  $x = y$ , then  $y < 2$ .

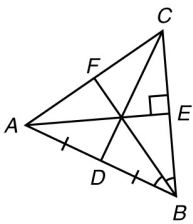
The perimeter of an isosceles triangle is 24 cm. If the length of one side is 6 cm, which of the following can be the lengths of the other two sides?

- I. 6 cm
- II. 9 cm
- III. 12 cm

- A. I only
- B. II only
- C. I and II only
- D. I, II and III

If the lengths of two sides of an isosceles triangle are 6 cm and 14 cm, what is the perimeter of the triangle?

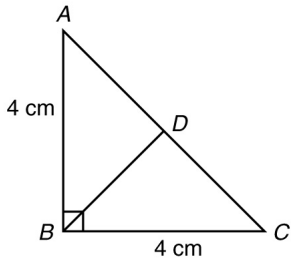
- A. 20 cm
- B. 26 cm
- C. 30 cm
- D. 34 cm



In the figure,  $\triangle ABC$  is an equilateral triangle,  $\angle AEC = 90^\circ$ ,  $\angle ABF = \angle CBF$  and  $AD = BD$ . Which of the following line segment(s) is/are median(s) of  $\triangle ABC$ ?

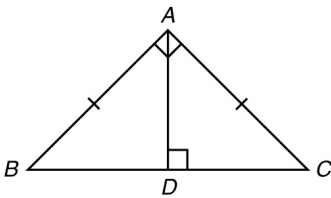
- I.  $AE$
- II.  $BF$
- III.  $CD$

- A. I only
- B. II only
- C. III only
- D. I, II and III



In the figure,  $\triangle ABC$  is a right-angled triangle with  $\angle B = 90^\circ$ . If  $AB = BC = 4$  cm and  $BD$  is the median of  $AC$ , find the length of  $BD$ ?

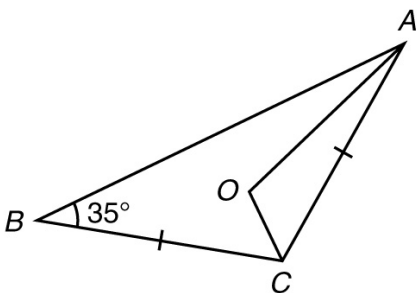
- A. 3 cm
- B. 4 cm
- C.  $2\sqrt{2}$  cm
- D.  $4\sqrt{2}$  cm



In the figure,  $\triangle ABC$  is a right-angled triangle with  $\angle A = 90^\circ$ . If  $AB = AC$  and  $AD$  is the altitude of  $BC$ , which of the following is/are true?

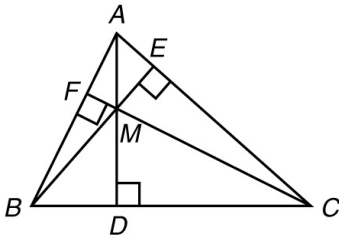
- I.  $\triangle ABD \cong \triangle ACD$
- II.  $BD = CD$
- III.  $AD = \frac{1}{2}BC$

- A. I only
- B. II only
- C. I and III only
- D. I, II and III



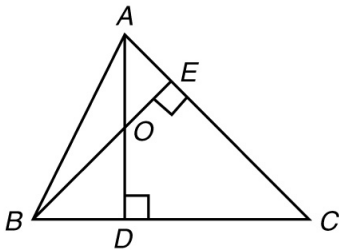
In the figure,  $\angle ABC = 35^\circ$ ,  $AC = BC$  and  $O$  is the incentre of  $\triangle ABC$ . Find  $\angle AOC$ .

- A.  $107.5^\circ$
- B.  $108^\circ$
- C.  $109.5^\circ$
- D.  $110^\circ$



In the figure,  $AD$ ,  $BE$  and  $CF$  are three altitudes of  $\triangle ABC$ . If they intersect at  $M$ , then  $M$  is the

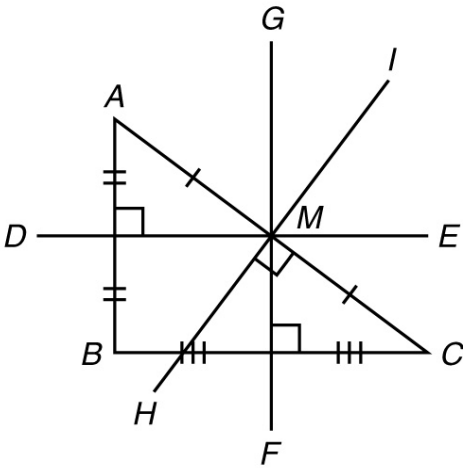
- A. incentre of  $\triangle ABC$ .
- B. orthocentre of  $\triangle ABC$ .
- C. circumcentre of  $\triangle ABC$ .
- D. centroid of  $\triangle ABC$ .



In the figure,  $AD$  and  $BE$  are two altitudes of  $\triangle ABC$ . Which of the following must be true?

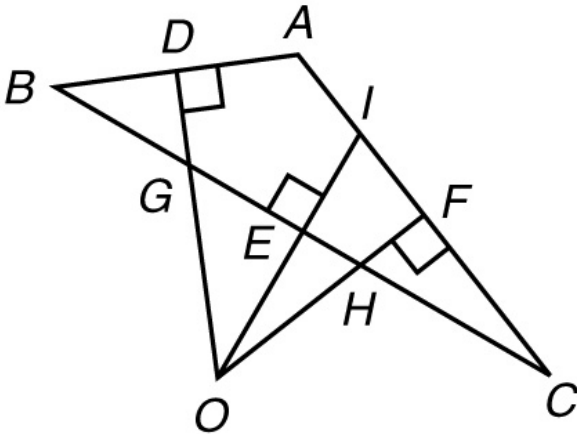
- I. If  $AD = CD$ ,  $BE = CE$ .
- II.  $\triangle ADC \sim \triangle BEC$
- III.  $O$  is called the incentre of  $\triangle ABC$ .

- A. II only
- B. I and II only
- C. II and III only
- D. I, II and III



In the figure,  $DE$ ,  $FG$  and  $HI$  are three perpendicular bisectors of  $\triangle ABC$ . If they intersect at  $M$ , then  $M$  is the

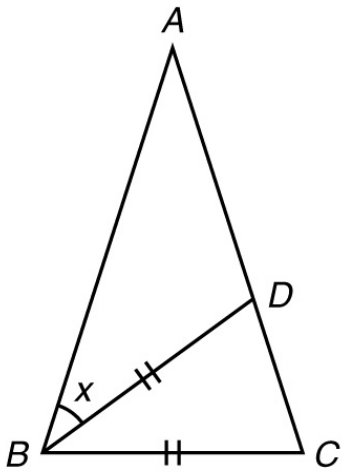
- A. incentre of  $\triangle ABC$ .
- B. centroid of  $\triangle ABC$ .
- C. circumcentre of  $\triangle ABC$ .
- D. orthocentre of  $\triangle ABC$ .



In the figure,  $OGD$ ,  $OEI$  and  $OHF$  are the three perpendicular bisectors of  $\triangle ABC$ . Which of the following must be true?

- I.  $O$  is the orthocentre of  $\triangle ABC$ .
- II.  $\triangle OFI \sim \triangle CFH$
- III.  $BE = CE$

- A. I only
- B. III only
- C. I and III only
- D. II and III only



In the figure,  $BC = BD$  and  $BD$  is the angle bisector of  $\angle ABC$  in  $\triangle ABC$ . If  $AB = AC$ , find  $x$ .

- A.  $32^\circ$
- C.  $40^\circ$
- B.  $36^\circ$
- D.  $44^\circ$