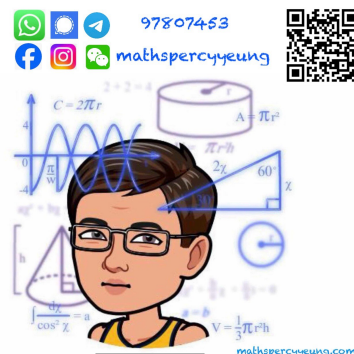
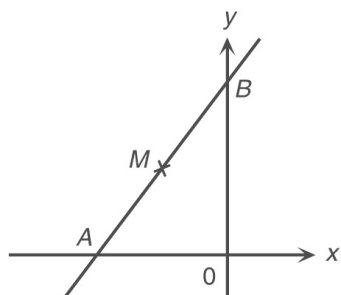
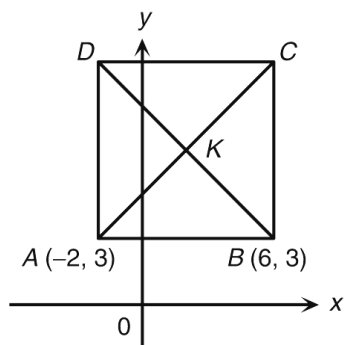


# Ch13 Coordinate Geometry of Straight Lines (13) Set 1

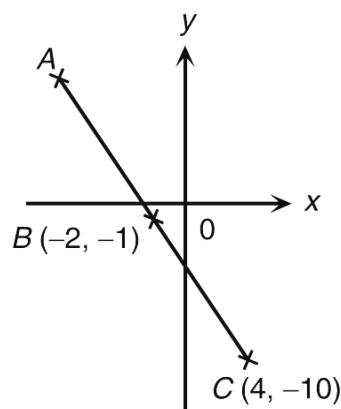
In the figure, a straight line cuts the  $x$ -axis and the  $y$ -axis at  $A(-6, 0)$  and  $B(0, 8)$  respectively. If  $M$  is the mid-point of  $AB$ , find the coordinates of  $M$ .



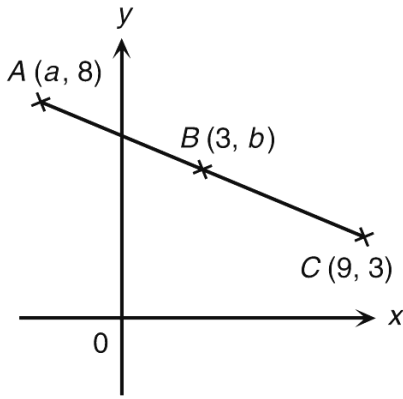
In the figure, the diagonals of the square  $ABCD$  intersect at  $K$ . Find the coordinates of  $K$ .



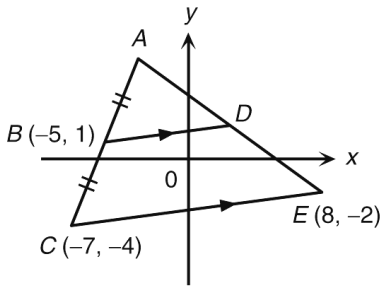
In the figure,  $ABC$  is a straight line and  $AB = BC$ . Find the coordinates of  $A$ .



In the figure,  $ABC$  is a straight line and  $AB = BC$ . Find the values of  $a$  and  $b$ .

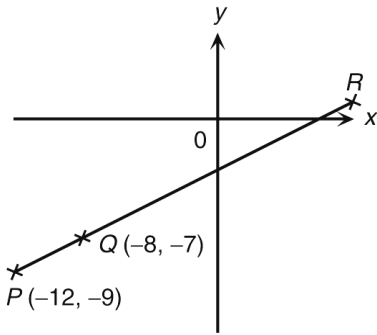


In the figure,  $B(-5, 1)$  is the mid-point of the line segment joining  $A$  and  $C(-7, -4)$ .  $D$  is a point on  $AE$  such that  $BD \parallel CE$ . Find the coordinates of  $A$  and  $D$ .

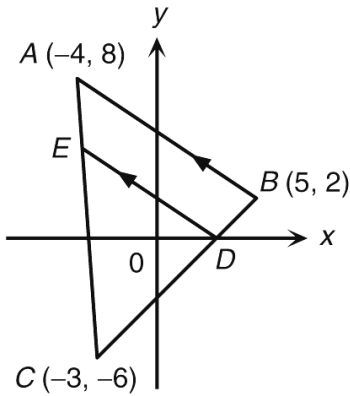


$P$  is a point lying on the line segment joining  $A(-3, -4)$  and  $B(7, 8)$ , where  $AP : PB = 3 : 2$ . Find the coordinates of  $P$ .

The figure shows two points  $P(-12, -9)$  and  $Q(-8, -7)$ . If  $R$  is a point on  $PQ$  produced such that  $PQ : QR = 1 : 4$ , find the coordinates of  $R$ .

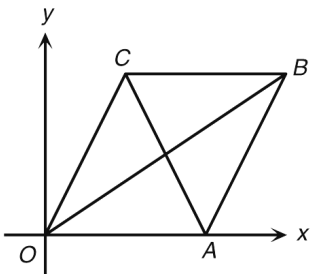


In the figure,  $BC$  cuts the  $x$ -axis at  $D$ .  $E$  is a point on  $AC$  such that  $BA \parallel DE$ .

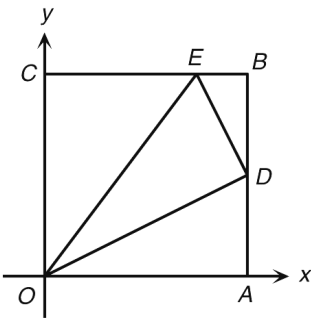


- (a) Find  $BD : DC$ .
- (b) Find the coordinates of  $E$ .

In the figure,  $OABC$  is a parallelogram. Prove by the analytic approach that  $OB^2 + AC^2 = 2(OA^2 + OC^2)$ .

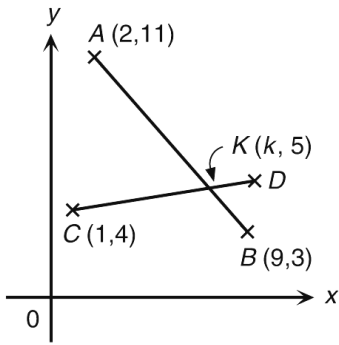


In the figure,  $OABC$  is a square.  $D$  is the mid-point of  $AB$ .  $E$  is a point on  $CB$  such that  $CE : EB = 3 : 1$ .



- (a) Let  $OA = a$ . Express the coordinates of  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  in terms of  $a$ .
- (b) (i) Hence, prove by the analytic approach that  $OE^2 = OD^2 + DE^2$ .
- (ii) State what kind of  $\triangle ODE$  is.

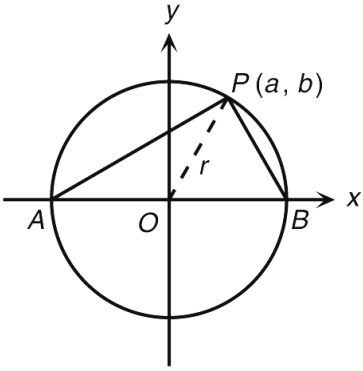
In the figure, the line joining  $A(2, 11)$  and  $B(9, 3)$  intersects the line joining  $C(1, 4)$  and  $D$  at  $K(k, 5)$ .



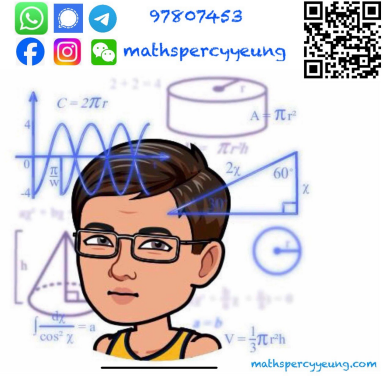
If  $AK : KB = CK : KD = m : n$ , find

- (a)  $m : n$ ,
- (b) the coordinates of  $D$ .

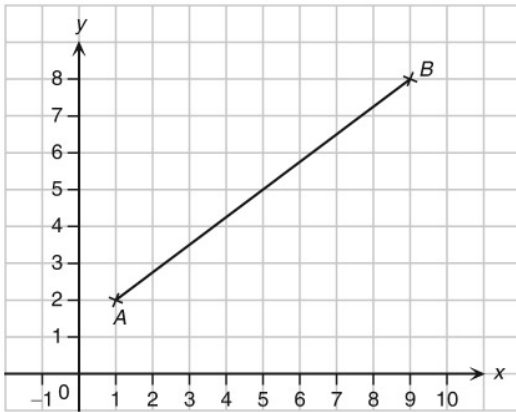
In the figure,  $O$  is the centre of the circle with radius  $r$  and  $P(a, b)$  is a point on the circle.



- (a) Prove that  $a^2 + b^2 = r^2$ .
- (b) Prove by the analytic approach that  $AP \perp PB$ .

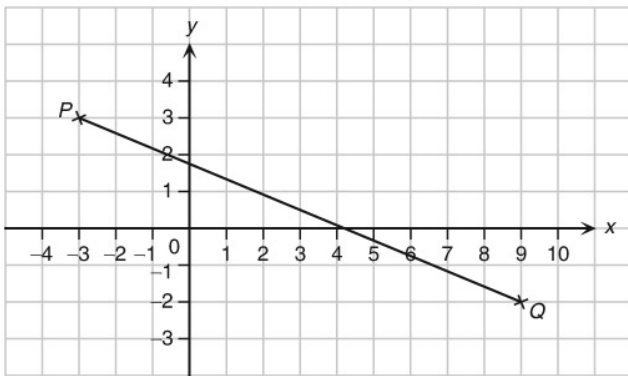


Refer to the figure.



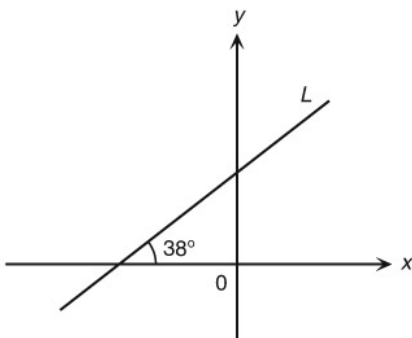
- (a) Find the length of  $AB$ .
- (b) Find the slope of  $AB$ .

Refer to the figure.



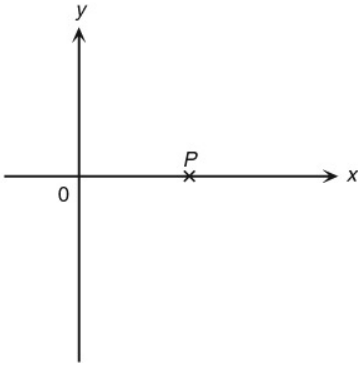
- (a) Find the length of  $PQ$ .
- (b) Find the slope of  $PQ$ .

The figure shows a straight line  $L$  with inclination  $38^\circ$ . Find the slope of  $L$  correct to 3 significant figures.

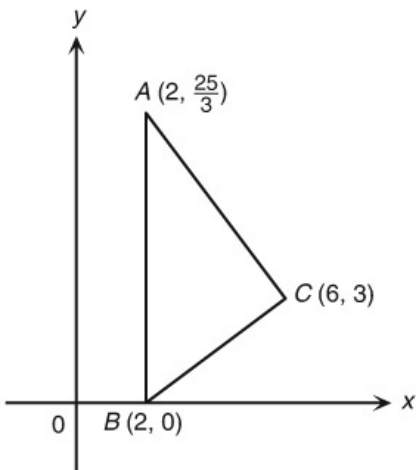


It is given that the slope of a straight line  $L$  is 1.

- (a) Find the inclination of  $L$ .
- (b) If  $L$  passes through a point  $P$  as shown below, draw the straight line  $L$  and mark its inclination in the figure.



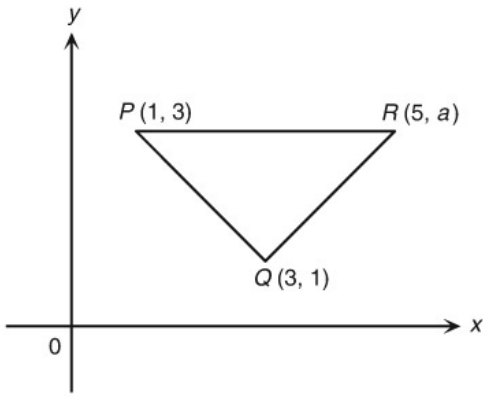
Refer to the figure.



- (a) Find the perimeter of  $\triangle ABC$ .
- (b) Show that  $\triangle ABC$  is a right-angled triangle.

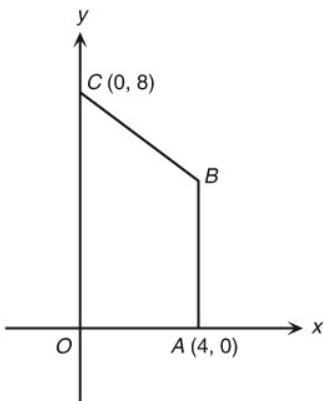


In the figure,  $P(1, 3)$ ,  $Q(3, 1)$  and  $R(5, a)$  are three points above the  $x$ -axis and  $PQ = QR$ .



- (a) Find the value of  $a$ .
- (b) Find the area of  $\triangle PQR$ .

In the figure,  $A(4, 0)$  and  $C(0, 8)$  are points on the  $x$ -axis and the  $y$ -axis respectively.  $B$  is a point such that  $AB \parallel OC$  and  $AB = CB$ .



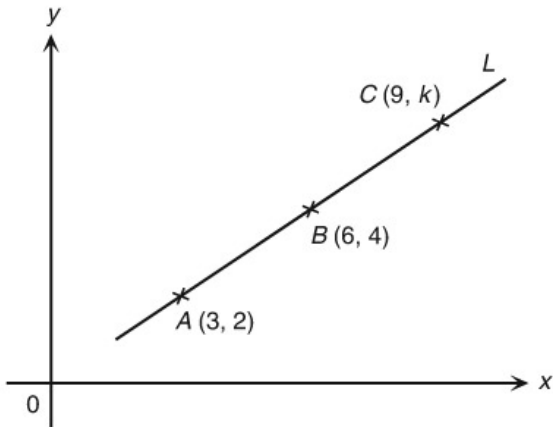
- (a) Find the coordinates of  $B$ .
- (b) Find the area of quadrilateral  $OABC$ .

In each of the following, determine whether the three points are collinear.

(a)  $A(-4, -1)$ ,  $B(-2, 1)$  and  $C(3, 6)$

(b)  $D(5, 2)$ ,  $E(8, -4)$  and  $F(10, -7)$

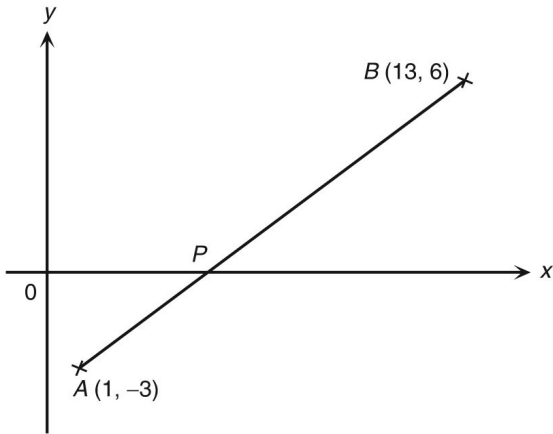
In the figure, the straight line  $L$  passes through  $A(3, 2)$ ,  $B(6, 4)$  and  $C(9, k)$ .



(a) Find the value of  $k$ .

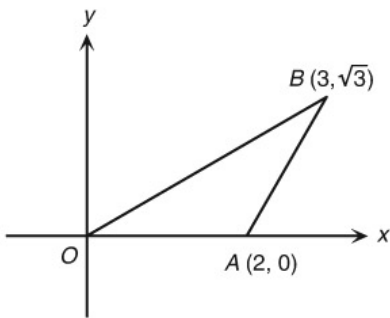
(b) Does  $P(12, 6)$  lie on the straight line  $L$ ? Explain your answer.

Referring to the figure,  $AB$  cuts the  $x$ -axis at  $P$ .



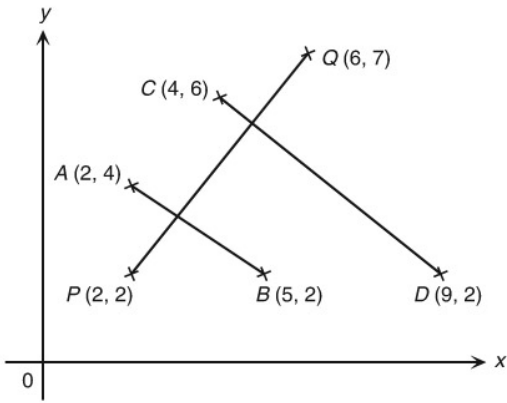
- (a) Find the coordinates of  $P$ .
- (b) Find  $AP : PB$ .

In the figure,  $A(2, 0)$  and  $B(3, \sqrt{3})$  are two points on a rectangular coordinate plane.

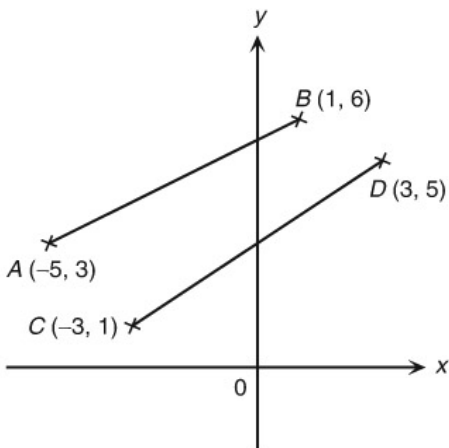


- (a) Find the slope and the inclination of
  - (i)  $OB$ ,
  - (ii)  $AB$ .
- (b) (i) Find  $\angle ABO$ .
- (ii) State what kind of triangle  $OAB$  is.

Referring to the figure, determine whether  $AB$  and  $CD$  are perpendicular to  $PQ$ .

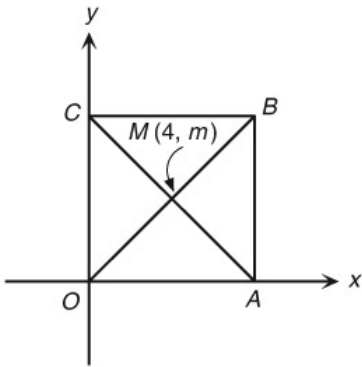


Refer to the figure.



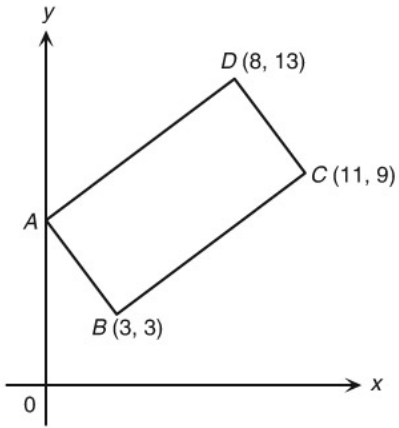
- (a) Show that  $AB$  and  $CD$  are not parallel.
- (b) If  $B$  is translated upwards by  $k$  unit(s) to  $B'$  such that  $AB' \parallel CD$ , find the value of  $k$ .

In the figure,  $A$  and  $C$  are points on the  $x$ -axis and the  $y$ -axis respectively.  $OABC$  is a square. Its diagonals  $OB$  and  $AC$  intersect at  $M(4, m)$ .



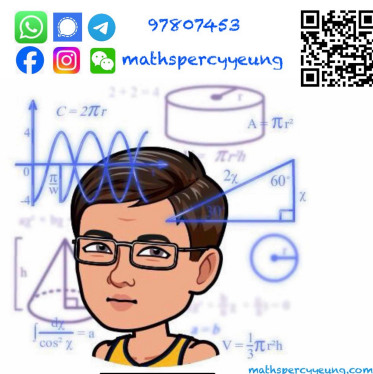
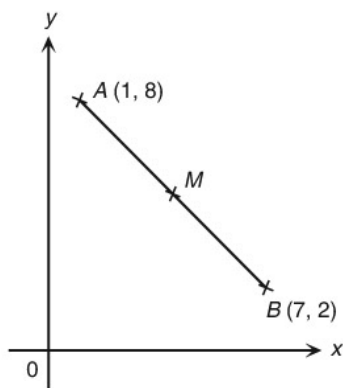
- (a) Find the value of  $m$ .
- (b) Find the coordinates of  $A$ ,  $B$  and  $C$ .

Referring to the figure,  $A$  is a point on the  $y$ -axis such that  $AB \parallel DC$ .

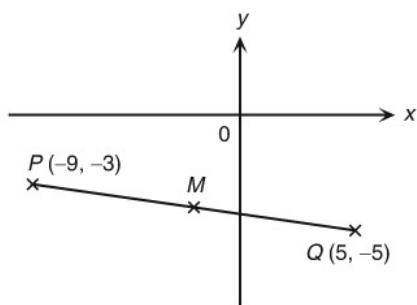


- (a) Find the coordinates of  $A$ .
- (b) Show that  $ABCD$  is a rectangle.

In the figure,  $M$  is the mid-point of the line segment joining  $A(1, 8)$  and  $B(7, 2)$ . Find the coordinates of  $M$ .



In the figure,  $P(-9, -3)$  and  $Q(5, -5)$  are the end points of the line segment  $PQ$ . If  $M$  is the mid-point of  $PQ$ , find the coordinates of  $M$ .



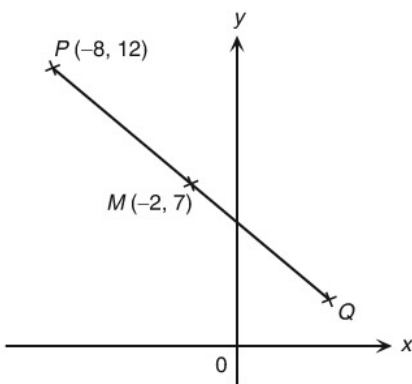
Given that a point  $P$  bisects the line segment joining  $A(-6, 7)$  and  $B(1, -3)$ , find the coordinates of  $P$ .

It is given that  $P$  is the mid-point of the line segment joining  $A(0, 10)$  and  $B(-8, 14)$ .

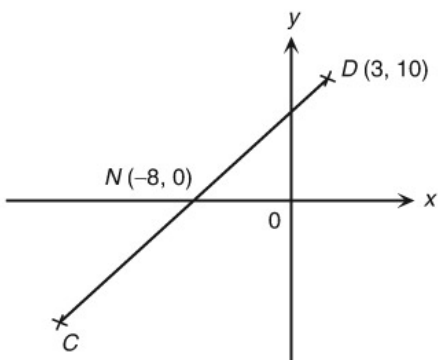
(a) Find the coordinates of  $P$ .

(b) If  $Q$  is the mid-point of  $PB$ , find the coordinates of  $Q$ .

Referring to the figure,  $M$  is the mid-point of  $PQ$ . Find the coordinates of  $Q$ .



Referring to the figure,  $N$  is the mid-point of  $CD$ . Find the coordinates of  $C$ .





Given that  $M(3, 6)$  is the mid-point of the line segment joining  $A(a, -4)$  and  $B(7, b)$ , find the values of  $a$  and  $b$ .

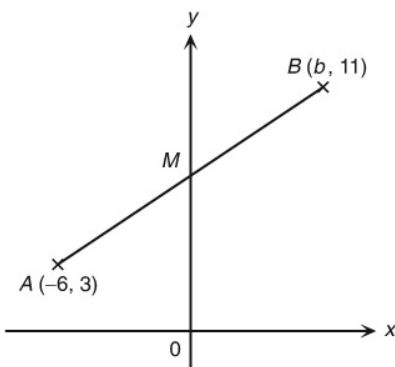
Given that a point  $P(7, -1)$  bisects the line segment joining  $C(2, c)$  and  $D(d, 4)$ , find the values of  $c$  and  $d$ .

Given that the line segment joining  $A(4, 5)$  and  $M(7, 9)$  is produced to a point  $B$  and  $AM = MB$ , find the coordinates of  $B$ .

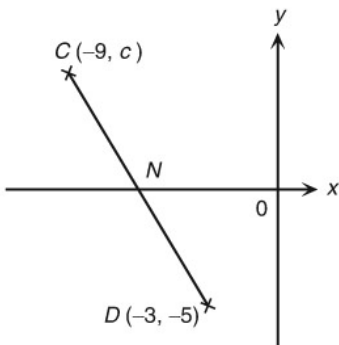
It is given that  $N(-8, 3)$  divides the line segment joining  $C$  and  $D(-4, -3)$  into 2 equal parts.

- (a) Find the coordinates of  $C$ .
- (b) If  $C$  is the mid-point of  $BN$ , find the coordinates of  $B$ .

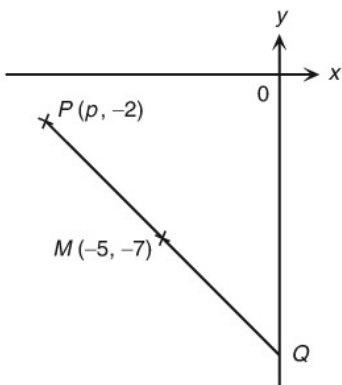
Referring to the figure, the mid-point  $M$  of  $AB$  lies on the  $y$ -axis. Find the value of  $b$  and the coordinates of  $M$ .



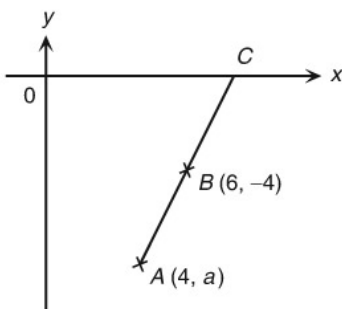
In the figure, the line segment joining  $C(-9, c)$  and  $D(-3, -5)$  cuts the  $x$ -axis at  $N$ , where  $CN = ND$ . Find the value of  $c$  and the coordinates of  $N$ .



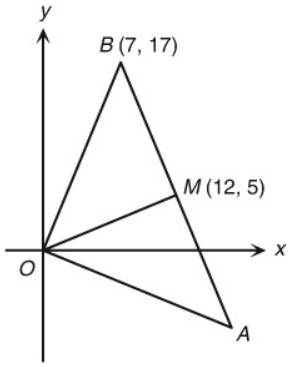
In the figure,  $Q$  is a point on the  $y$ -axis.  $M(-5, -7)$  is the mid-point of the line segment joining  $P(p, -2)$  and  $Q$ . Find the value of  $p$  and the coordinates of  $Q$ .



Referring to the figure,  $AB$  is produced to meet the  $x$ -axis at  $C$  and  $AB = BC$ . Find the value of  $a$  and the coordinates of  $C$ .

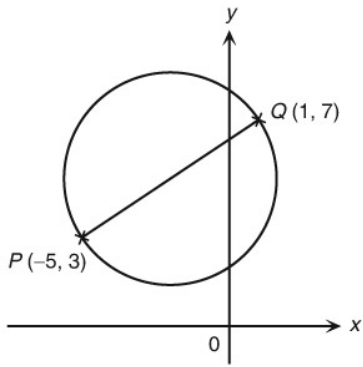


Referring to the figure,  $OM$  is the perpendicular bisector of  $AB$  in  $\triangle OAB$ .



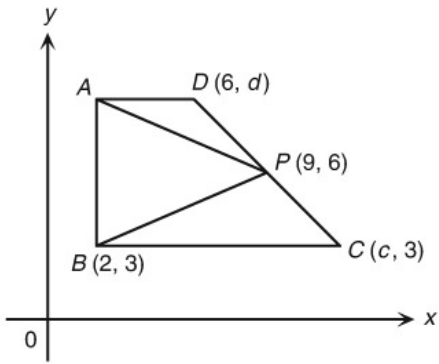
- (a) Find the coordinates of  $A$ .
- (b) Find the area of  $\triangle OAB$ .

Referring to the figure,  $PQ$  is a diameter of a circle.



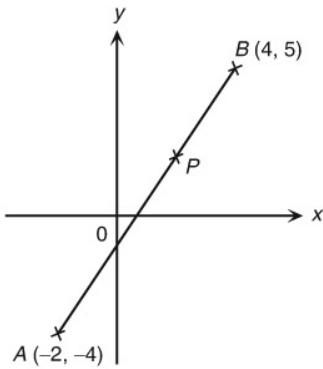
- (a) Find the coordinates of the centre  $C$  of the circle.
- (b) It is given that  $R(-5, 7)$  and  $S$  are the end points of the diameter  $RS$  of the circle. Find the coordinates of  $S$ .

Referring to the figure,  $ABCD$  is a right-angled trapezium, where  $AD \parallel BC$  and  $\angle ABC = 90^\circ$ .  $P$  is the mid-point of  $CD$ .

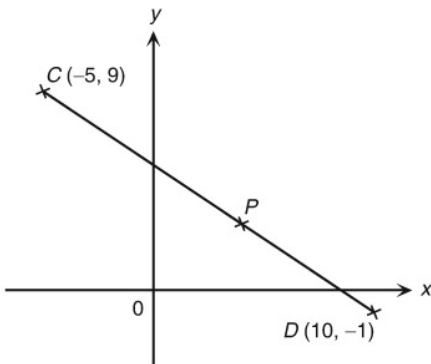


- (a) Find the values of  $c$  and  $d$ .
- (b) Find the coordinates of  $A$ .
- (c) Find the ratio of the area of  $\triangle ABP$  to that of trapezium  $ABCD$ .

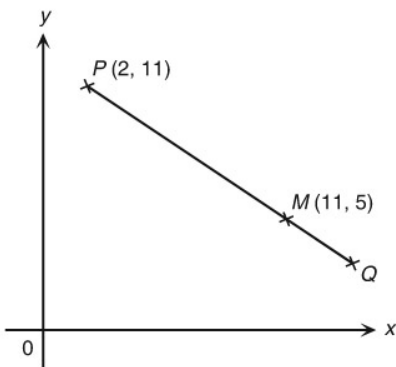
The figure shows two points  $A(-2, -4)$  and  $B(4, 5)$ .  $P$  lies on  $AB$  such that  $AP : PB = 2 : 1$ . Find the coordinates of  $P$ .



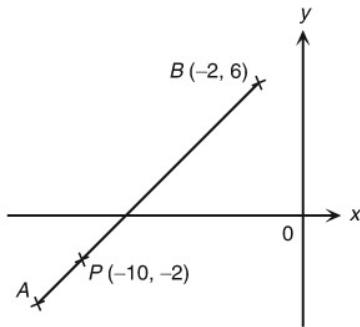
In the figure,  $P$  is a point on the line segment joining  $C(-5, 9)$  and  $D(10, -1)$  such that  $CP : PD = 3 : 2$ . Find the coordinates of  $P$ .



In the figure, the coordinates of an end point  $P$  of the line segment  $PQ$  are  $(2, 11)$ .  $M(11, 5)$  lies on  $PQ$  such that  $PM : MQ = 3 : 1$ . Find the coordinates of  $Q$ .



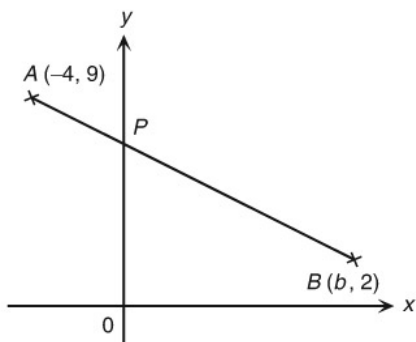
Referring to the figure,  $P$  is a point on  $AB$  such that  $AP : PB = 1 : 4$ . Find the coordinates of  $A$ .



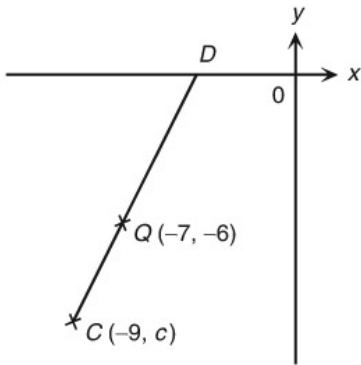
Given that  $Q(0, 4)$  divides the line segment joining  $P(-10, -1)$  and  $R(2, 5)$  into two parts, find  $PQ : QR$ .

Given that  $T(6, 2)$  lies on the line segment joining  $P(8, -5)$  and  $Q\left(\frac{36}{7}, 5\right)$ , find  $PT : TQ$ .

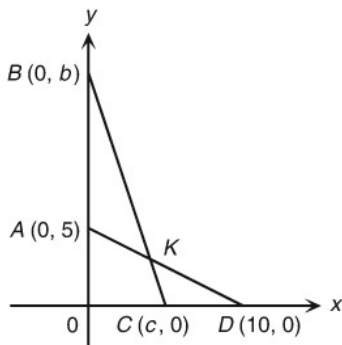
In the figure, the line segment joining  $A(-4, 9)$  and  $B(b, 2)$  cuts the  $y$ -axis at  $P$  and  $AP : PB = 2 : 5$ . Find the value of  $b$  and the coordinates of  $P$ .



Referring to the figure,  $CQ$  is produced to meet the  $x$ -axis at  $D$  and  $CQ : QD = 2 : 3$ . Find the value of  $c$  and the coordinates of  $D$ .



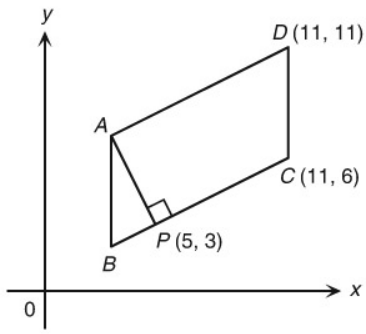
Referring to the figure,  $AD$  and  $BC$  intersect at  $K$ , where  $AK : KD = 2 : 3$  and  $BK : KC = 4 : 1$ .



- (a) Find the coordinates of  $K$ .
- (b) Find the values of  $b$  and  $c$ .
- (c) Consider the line segments  $AK$ ,  $KD$ ,  $BK$  and  $KC$ . Which one is the shortest?

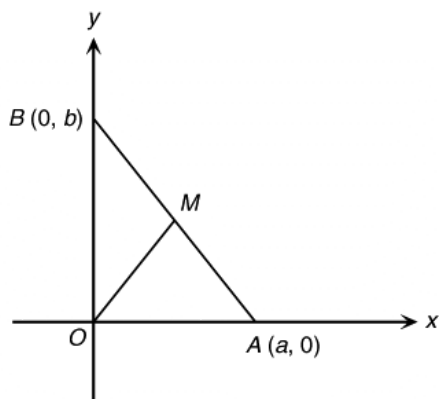


Referring to the figure,  $ABCD$  is a parallelogram.  $P$  is a point on  $BC$  such that  $AP \perp BC$  and  $BP : PC = 1 : 3$ .

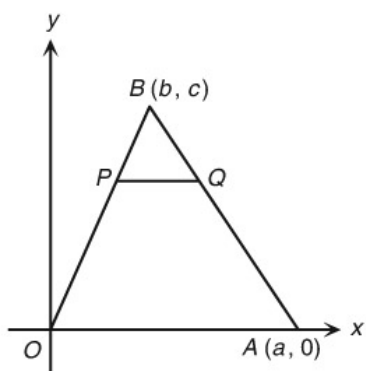


- (a) Find the coordinates of  $A$  and  $B$ .
- (b) Find the area of parallelogram  $ABCD$ .

In the figure, the coordinates of  $A$  and  $B$  are  $(a, 0)$  and  $(0, b)$  respectively. If  $M$  is the mid-point of  $AB$ , prove that  $OM = \frac{1}{2} AB$ .

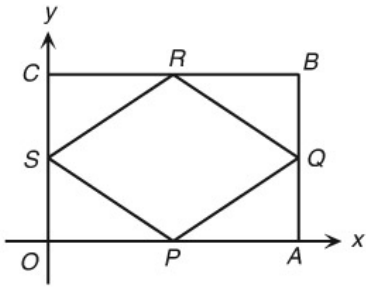


In the figure, the coordinates of  $A$  and  $B$  are  $(a, 0)$  and  $(b, c)$  respectively.  $P$  and  $Q$  are points on  $OB$  and  $AB$  respectively such that  $OP : PB = AQ : QB = 2 : 1$ .

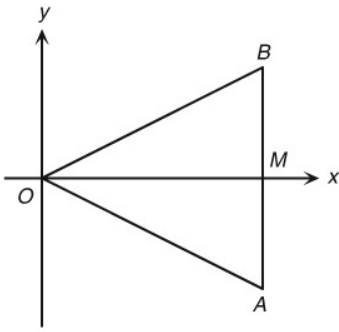


- (a) Express the coordinates of  $P$  and  $Q$  in terms of  $a, b$  and  $c$ .
- (b) (i) Prove that  $PQ \parallel OA$ .
- (ii) Prove that  $PQ = \frac{1}{3} OA$ .

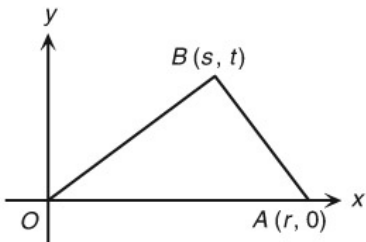
In the figure,  $OABC$  is a rectangle, where  $OA = a$  units and  $OC = c$  units.  $P$ ,  $Q$ ,  $R$  and  $S$  are the mid-points of  $OA$ ,  $AB$ ,  $BC$  and  $OC$  respectively. Prove that  $PQRS$  is a rhombus.



In the figure,  $M$  is a point on the  $x$ -axis.  $OM$  is the perpendicular bisector of  $AB$  in  $\triangle OAB$ . Prove that  $\triangle OAB$  is an isosceles triangle by the analytic approach.

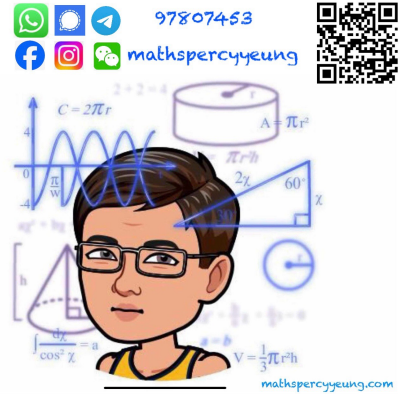
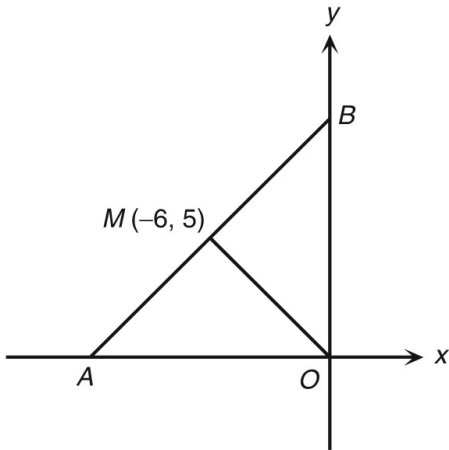


In the figure, the coordinates of  $A$  and  $B$  are  $(r, 0)$  and  $(s, t)$  respectively. If  $OA^2 = OB^2 + AB^2$ , prove that  $\angle OBA = 90^\circ$  by the analytic approach.



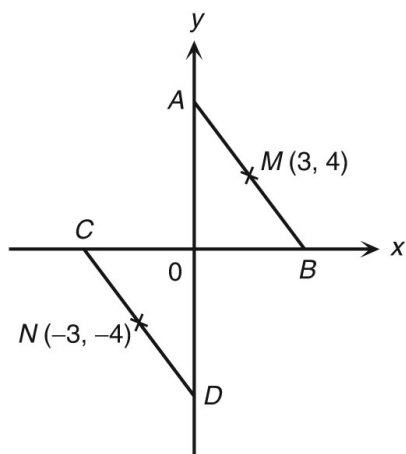
Ch13 Coordinate Geometry of Straight Lines (T1) Set 4

In the figure,  $A$  and  $B$  are points on the  $x$ -axis and the  $y$ -axis respectively.  $M(-6, 5)$  is the mid-point of  $AB$ .



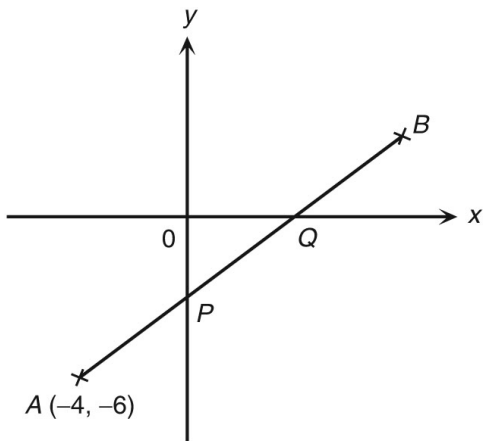
- (a) Find the coordinates of  $A$  and  $B$ .
- (b) Is  $OM$  perpendicular to  $AB$ ? Explain your answer.

In the figure,  $A$  and  $D$  are two points on the  $y$ -axis.  $B$  and  $C$  are two points on the  $x$ -axis.  $M(3, 4)$  and  $N(-3, -4)$  are the mid-points of  $AB$  and  $CD$  respectively.



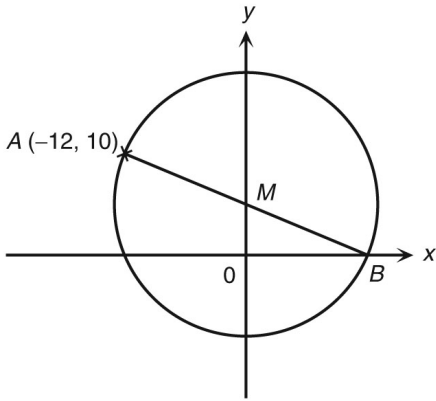
- (a) Find the coordinates of  $A$ ,  $B$ ,  $C$  and  $D$ .
- (b) Determine whether  $AB$  and  $CD$  are parallel.

Referring to the figure,  $P$  and  $Q$  are points on the  $y$ -axis and the  $x$ -axis respectively.  $P$  and  $Q$  divide  $AB$  into 3 equal parts.



- (a) Find the coordinates of  $P$ ,  $Q$  and  $B$ .
- (b) Find the length of  $AB$ .

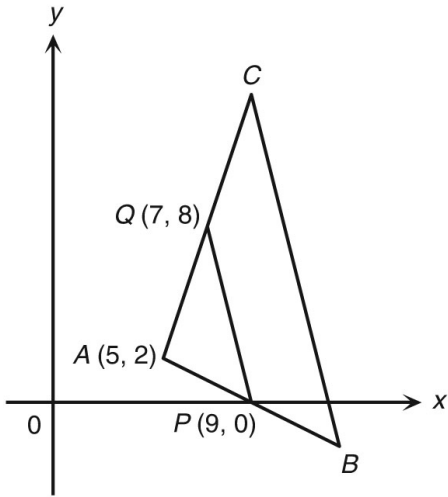
Referring to the figure,  $B$  and  $M$  are points on the  $x$ -axis and the  $y$ -axis respectively.  $AB$  is a diameter of a circle with centre  $M$ .



- (a) Find the coordinates of  $M$  and  $B$ .
- (b) If  $P$  is a point on the negative  $x$ -axis such that  $\angle APB = 90^\circ$ , show that  $P$  lies on the circle.

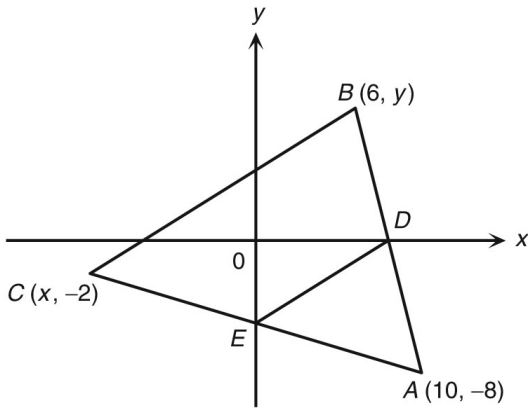


Referring to the figure,  $P$  and  $Q$  are the mid-points of  $AB$  and  $AC$  respectively.



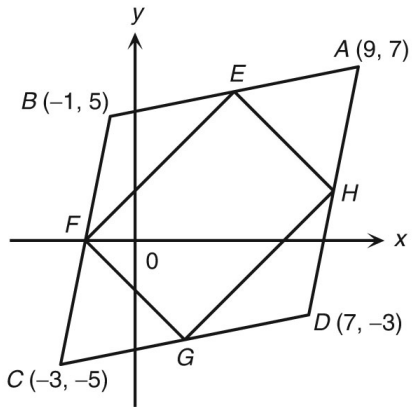
- (a) Find the coordinates of  $B$  and  $C$ .
- (b) (i) Find the slopes of  $PQ$  and  $BC$ .
- (ii) Hence, determine whether  $PQ$  is parallel to  $BC$ .

In the figure, the line segment joining  $A(10, -8)$  and  $B(6, y)$  cuts the  $x$ -axis at  $D$ , where  $AD = DB$ . The line segment joining  $A$  and  $C(x, -2)$  cuts the  $y$ -axis at  $E$ , where  $AE = EC$ .



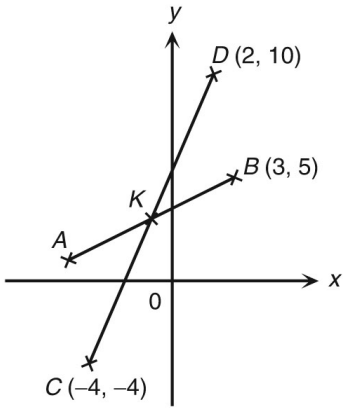
- (a) (i) Find the value of  $x$  and the coordinates of  $E$ .  
(ii) Find the value of  $y$  and the coordinates of  $D$ .
- (b) Show that  $DE = \frac{1}{2}BC$ .

In the figure,  $A(9, 7)$ ,  $B(-1, 5)$ ,  $C(-3, -5)$  and  $D(7, -3)$  are the vertices of a quadrilateral.  $E$ ,  $F$ ,  $G$  and  $H$  are the mid-points of  $AB$ ,  $BC$ ,  $CD$  and  $DA$  respectively.

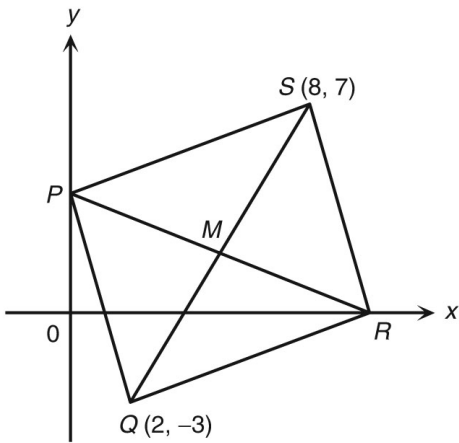


- (a) Find the coordinates of  $E$ ,  $F$ ,  $G$  and  $H$ .
- (b) Show that  $EFGH$  is a rectangle.

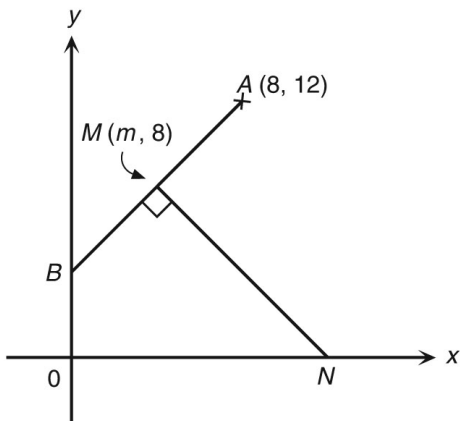
Referring to the figure,  $K$  bisects both the line segments  $AB$  and  $CD$ . Find the coordinates of  $A$  and  $K$ .



Referring to the figure,  $PQRS$  is a parallelogram and its diagonals intersect at  $M$ . Find the coordinates of  $P$ ,  $M$  and  $R$ .

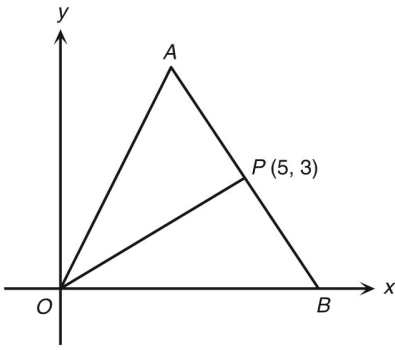


Referring to the figure,  $AM$  is produced to meet the  $y$ -axis at  $B$  and  $AM = MB$ .  $N$  is a point on the  $x$ -axis such that  $NM \perp AB$ .

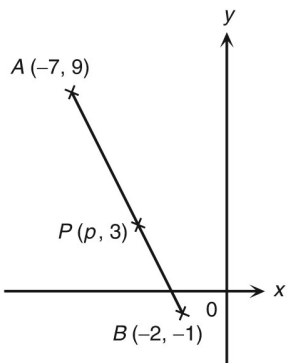


- (a) Find the value of  $m$  and the coordinates of  $B$ .
- (b) Find the coordinates of  $N$ .
- (c) If  $C$  and  $D$  are points on the  $x$ -axis such that  $AC \parallel MN \parallel BD$ ,
  - (i) find the coordinates of  $C$  and  $D$ ,
  - (ii) determine whether  $N$  is the mid-point of  $DC$ .

In the figure, the coordinates of  $P$  are  $(5, 3)$  and  $B$  lies on the  $x$ -axis.  $OP$  is the median of  $AB$  in  $\triangle AOB$  and its area is 21 sq. units. Find the coordinates of  $A$  and  $B$ .

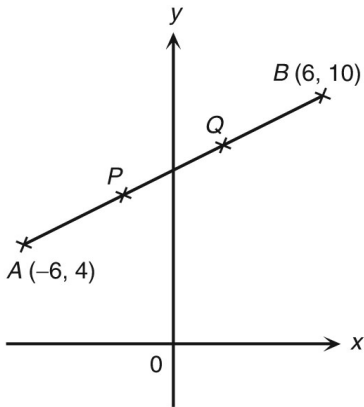


In the figure, the coordinates of  $A$  and  $B$  are  $(-7, 9)$  and  $(-2, -1)$  respectively.  $P(p, 3)$  is a point on  $AB$ .

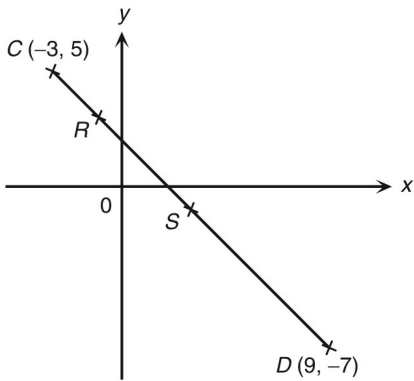


- (a) Find  $AP : PB$ .
- (b) Hence, find the value of  $p$ .

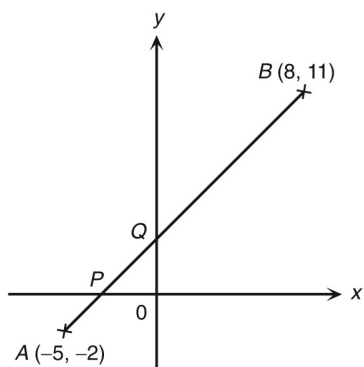
In the figure, the coordinates of  $A$  and  $B$  are  $(-6, 4)$  and  $(6, 10)$  respectively.  $P$  and  $Q$  divide  $AB$  into 3 equal parts. Find the coordinates of  $P$  and  $Q$ .



In the figure, the coordinates of  $C$  and  $D$  are  $(-3, 5)$  and  $(9, -7)$  respectively.  $R$  and  $S$  are two points on  $CD$  such that  $CR : RS : SD = 1 : 2 : 3$ . Find the coordinates of  $R$  and  $S$ .



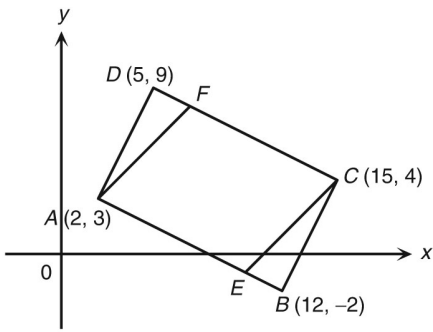
In the figure, the line segment joining  $A(-5, -2)$  and  $B(8, 11)$  cuts the  $x$ -axis and the  $y$ -axis at  $P$  and  $Q$  respectively.



- (a) (i) Find  $AQ : QB$ .  
(ii) Hence, find the coordinates of  $Q$ .
- (b) (i) Find  $AP : PQ$ .  
(ii) Hence, find the coordinates of  $P$ .
- (c) Find  $AP : PQ : QB$ .

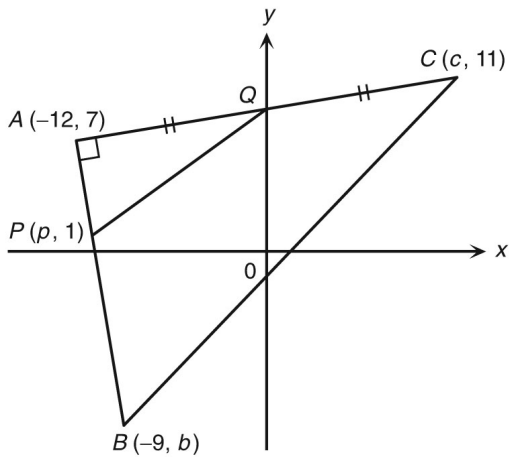


In the figure,  $A(2, 3)$ ,  $B(12, -2)$ ,  $C(15, 4)$  and  $D(5, 9)$  are the vertices of a quadrilateral.  $E$  and  $F$  are points on  $AB$  and  $CD$  respectively, such that  $AE : EB = CF : FD = 4 : 1$ .



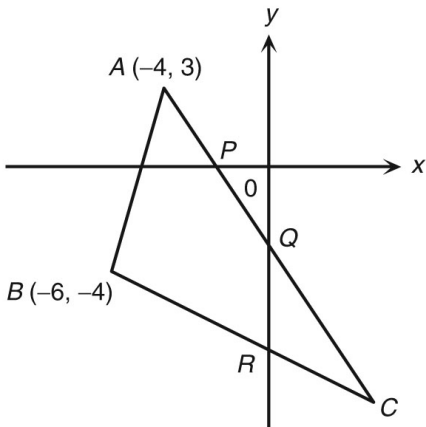
- (a)** Find the coordinates of  $E$  and  $F$ .  
**(b)** Show that  $AECF$  is a parallelogram.

Referring to the figure,  $AC$  cuts the  $y$ -axis at  $Q$  and  $AQ = QC$ .  $P$  lies on  $AB$  and  $AB \perp AC$ .

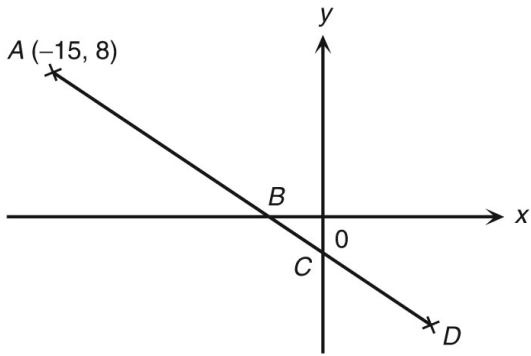


- (a) Find the value of  $c$  and the coordinates of  $Q$ .
- (b) Find the values of  $b$  and  $p$ .
- (c) Find  $AP : PB$ .

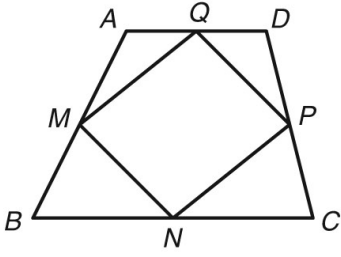
- In the figure,  $A(-4, 3)$ ,  $B(-6, -4)$  and  $C$  are the vertices of a triangle.  $AC$  cuts the  $x$ -axis and the  $y$ -axis at  $P$  and  $Q$  respectively.  $BC$  cuts the  $y$ -axis at  $R$ . If  $AP : PC = 1 : 3$  and  $BR : RC = 3 : 2$ , find the coordinates of
- (a)  $C$ ,
  - (b)  $P$ ,  $Q$  and  $R$ .



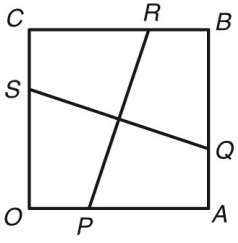
In the figure, the line segment joining  $A(-15, 8)$  and  $D$  cuts the  $x$ -axis at  $B$  and the  $y$ -axis at  $C$ . If  $AB : BC : CD = 4 : 1 : 2$ , find the coordinates of  $B$ ,  $C$  and  $D$ .



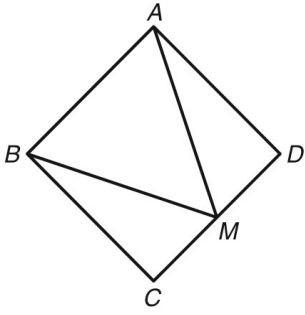
In the figure,  $ABCD$  is a trapezium, where  $AD \parallel BC$ .  $M$ ,  $N$ ,  $P$  and  $Q$  are the mid-points of  $AB$ ,  $BC$ ,  $CD$  and  $DA$  respectively. Prove that  $MNPQ$  is a parallelogram by the analytic approach.



It is given that  $OABC$  is a square of side  $a$  units.  $P$ ,  $Q$ ,  $R$  and  $S$  are points on  $OA$ ,  $AB$ ,  $BC$  and  $CO$  respectively, such that  $OP : PA = AQ : QB = BR : RC = CS : SO = 1 : 2$ . Prove that  $PR$  and  $QS$  bisect each other by the analytic approach.

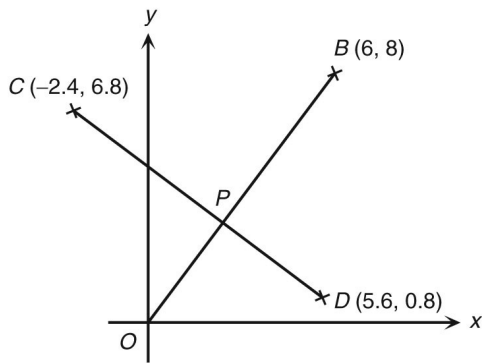


In the figure,  $ABCD$  is a rhombus.  $M$  is the mid-point of  $CD$ . If  $AM = BM$ , prove that  $ABCD$  is a square by the analytic approach.

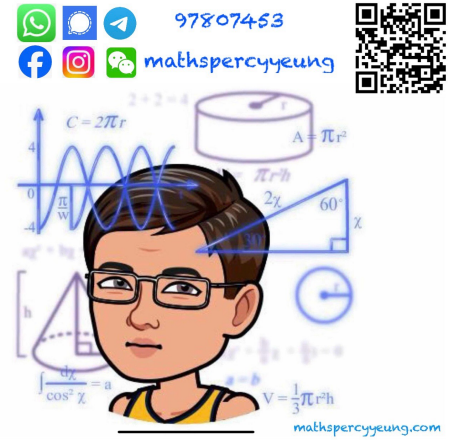


*Ch13 Coordinate Geometry of Straight Lines (TJ) Set 5*

In the figure, the coordinates of  $B$ ,  $C$  and  $D$  are  $(6, 8)$ ,  $(-2.4, 6.8)$  and  $(5.6, 0.8)$  respectively.  $OB$  and  $CD$  intersect at  $P$ , where  $OP : PB = 1 : r$  and  $CP : PD = s : 1$ .

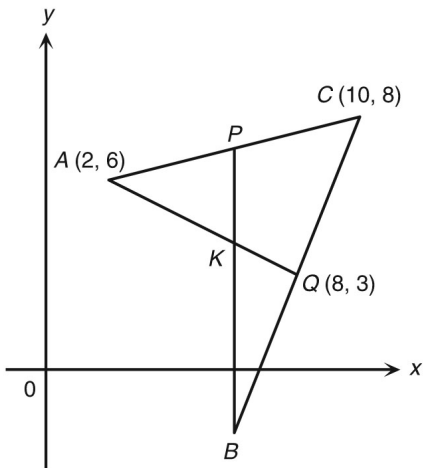


- (a) Find the values of  $r$  and  $s$ .
- (b) Show that  $OP = PD$  and  $CP = PB$ .



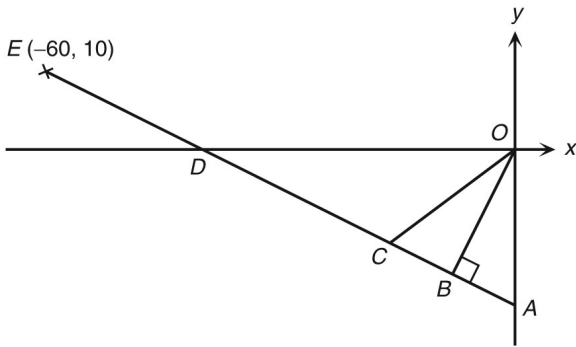


In the figure,  $P$  is the mid-point of the line segment joining  $A(2, 6)$  and  $C(10, 8)$ .  
 $Q(8, 3)$  bisects the line segment  $BC$ .  $AQ$  and  $BP$  intersect at  $K$ .



- (a) Find the coordinates of  $B$ ,  $K$  and  $P$ .  
(b) Find the area of quadrilateral  $CPKQ$ .

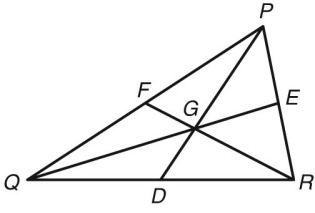
In the figure,  $A$  and  $D$  are points on the  $y$ -axis and the  $x$ -axis respectively.  $B$  and  $C$  lie on  $AD$ , such that  $OB \perp AD$  and  $OB$  is the angle bisector of  $\angle AOC$ .  $AD$  is produced to  $E(-60, 10)$  and  $AD : DE = 2 : 1$ .



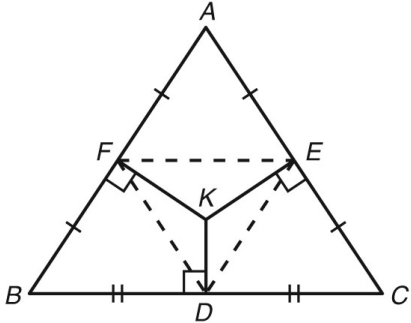
- (a) Find the coordinates of  $A$ ,  $B$ ,  $C$  and  $D$ .
- (b) Find  $AB : BC : CD : DE$ .

Prove that the three altitudes of an acute-angled triangle intersect at one point by the analytic approach.

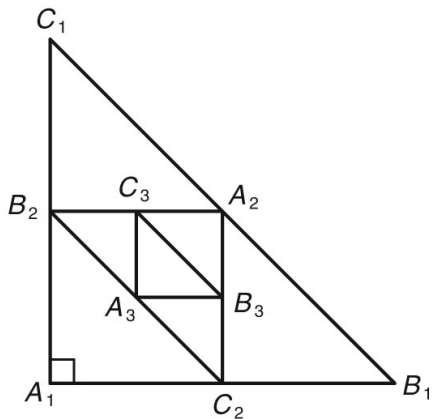
In the figure,  $G$  is the centroid of  $\triangle PQR$ . Prove that  $PG : GD = QG : GE = RG : GF = 2 : 1$  by the analytic approach.



In the figure,  $\triangle ABC$  is an isosceles triangle, where  $AB = AC$ .  $K$  is the circumcentre of  $\triangle ABC$ . Prove that  $K$  is the orthocentre of  $\triangle DEF$  by the analytic approach.



In the figure,  $\triangle A_1B_1C_1$  is a right-angled isosceles triangle, where  $\angle A_1 = 90^\circ$  and  $A_1B_1 = A_1C_1 = k$  units.  $\triangle A_2B_2C_2$  is formed by joining the mid-points of the sides of  $\triangle A_1B_1C_1$ .  $\triangle A_3B_3C_3$  is formed by joining the mid-points of the sides of  $\triangle A_2B_2C_2$ .



- (a) Prove that  $\triangle A_2B_2C_2$  and  $\triangle A_3B_3C_3$  are also right-angled isosceles triangles by the analytic approach.
- (b) Find the areas of  $\triangle A_1B_1C_1$ ,  $\triangle A_2B_2C_2$  and  $\triangle A_3B_3C_3$  in terms of  $k$ .
- (c) It is given that a fourth right-angled isosceles triangle  $A_4B_4C_4$  is formed by joining the mid-points of the sides of  $\triangle A_3B_3C_3$ , and this process is continued to form an infinite number of right-angled isosceles triangles. Find the areas of  $\triangle A_4B_4C_4$  and  $\triangle A_5B_5C_5$  in terms of  $k$ .  
(Hint: The areas of the triangles form a sequence with a pattern.)